DYNAMIC ECONOMETRIC MODELS
Vol. 10 – Nicolaus Copernicus University – Toruń – 2010

Jacek Kwiatkowski
Nicholas Copernicus University in Toruń

Unobserved Component Model for Forecasting Polish Inflation†

A b s t r a c t. This paper aims to use the local level models with GARCH and SV errors to predict Polish inflation. The series to be forecast, measured monthly, is consumer price index (CPI) in Poland during 1992-2008. We selected three forecasting models i.e. LL-GARCH(1,1) with Normal or Student errors and LL-SV. A simple AR(2)-SV model is used as a benchmark to assess the accuracy of prediction. The presented results indicate, that there is no clear advantage of LL models in forecasting Polish inflation over standard AR(2)-SV model, although all the models give satisfactory results.

K e y w o r d s: local level model, inflation, conditional heteroscedasticity.

1. Introduction

Econometric models, both of the structural and a-theoretical ones, are widely used to provide forecasts of inflation. In a recent study Stock and Watson (2007), found that a local level model with stochastic volatility gives the most accurate forecasts of quarterly inflation in the United States. In their paper they compare the accuracy of inflation forecasts of wide class of models including standard ARIMA time series models, time-varying parameters models (TVP) and the Phillips curve-based models.

In this paper, we examine several types of inflation forecasts in Poland, which are based on time-varying parameters model and subject them to tests for accuracy.

The paper is organized as follows. Section 2 presents the models which are used to forecast monthly inflation in Poland: LL-SV and LL-GARCH with two

† This work was financed from the Polish science budget resources in the years 2009-2011 as the research project no. N N111 431737.

I would like to thank Grzegorz Szafrański and one anonymous referee for helpful comments and suggestions; *e-mail: jkwiat@umk.pl*.

different distributions of the disturbances (Normal, Student's t) and standard AR(2)-SV model. In section 3, we compare forecast accuracy of the mentioned above models. We use two forecast accuracy measures, namely: the sign test and Wilcoxon signed rank test (see Diebold and Mariano, 1995). The predictive distribution calculated for the future observables enable us to provide a detailed analysis of the inflation forecasts, therefore we also present the predictive medians and interquartile range. It is also well known, that Polish monetary authorities conduct the policy under inflation targeting regime between 1.5% and 3.5% and the inflation prediction is one of the inputs to the Monetary Policy Council's decision-making process. Then, it is worth to consider what the posterior probability for the hypothesis is that inflation will stay inside the targeting bound and how this posterior probability changes as the forecasts horizon grows. Section 4 concludes the paper.

2. Unobserved Component Model with Time-Varying Conditional Variance

Stock and Watson (2007) used unobserved component model (LL-SV), which is very effective for forecasting inflation:

$$y_t = \delta_t + \varepsilon_t , \quad \varepsilon_t \sim N\left(0, \sigma_t^2\right), \; t = 1,2,\dots,T \tag{1}$$

$$\delta_t = \delta_{t-1} + \eta_t , \quad \eta_t \sim N\left(0, \omega_t^2\right), \tag{2}$$

where the irregular and level disturbances, ε_t and η_t, respectively, are mutually independent, δ_t denotes underlying stochastic level.

Consider now that ε_t and η_t are stationary SV processes, where:

$$h_{irreg,t} = \rho_{irreg} h_{irreg,t-1} + \zeta_{irreg,t} , \tag{3}$$

$$h_{level,t} = \rho_{level} h_{level,t-1} + \zeta_{level,t} , \tag{4}$$

and $\varepsilon_t \sim N\left(0, \sigma_t^2\right)$, $\eta_t \sim N\left(0, \omega_t^2\right)$, $\sigma_t^2 = \exp\left(h_{irreg,t}\right)$, $\omega_t^2 = \exp\left(h_{level,t}\right)$, $\rho_{irreg(level)} \in \left(-1,1\right)$ and $\zeta_{irreg(level)} \sim N\left(0, \gamma_{irreg(level)}^2\right)$.

It's easy to see that the reduced form of unobserved component model (1)-(2) is a local level model (LL) with restrictions in parameters: $\sigma_t^2 = \sigma^2$ and $\omega_t^2 = \omega^2$. The local level model is a well-known model and it has a long tradition in economic time series. The literature that considers its properties is very extensive and previously interested many authors (see for example Muth, 1960; Harvey, 1989; West and Harrison, 1989; Durbin and Koopman, 2001; Koop, 2003).

Consider the LL model when the disturbances follow Normal GARCH or Student-t GARCH. Assuming that each noise is a conditionally Normal process, we have:

$$\varepsilon_t \sim N\left(0, \sigma_t^2\right) \text{ and } \eta_t \sim N\left(0, \omega_t^2\right). \tag{5}$$

The equivalent Student-t disturbances are denoted as:

$$\varepsilon_t \sim t\left(0, \frac{v_{irreg}}{\left(v_{irreg} - 2\right)\sigma_t^2}, v_{irreg}\right) \text{ and } \eta_t \sim t\left(0, \frac{v_{level}}{\left(v_{level} - 2\right)\omega_t^2}, v_{level}\right), \tag{6}$$

where $t(a, P, v)$ denotes Student-t density with expectation a, precision P and v degrees of freedom.

For GARCH(1,1) process, the variance of the observation equation (1) and state equation (2) varies over time according to (see Bos, 2001):

$$\sigma_t^2 = h_{irreg,t} \text{ and } \omega_t^2 = h_{level,t}, \tag{7}$$

$$h_{irreg,t} = b_{1,irreg} h_{irreg,t-1} + a_{0,irreg} + a_{1,irreg}\left(E\varepsilon_{t-1}\right)^2, \tag{8}$$

$$h_{level,t} = b_{1,level} h_{level,t-1} + a_{0,\,level} + a_{1,\,level}\left(E\eta_{t-1}\right)^2, \tag{9}$$

with the parametric constraints that are sufficient for positivity and stationarity of the conditional variance:

$$a_{0,irreg(level)} \equiv 1 - b_{1,irreg(level)} - a_{1,irreg(level)},$$

$$b_{1,irreg(level)} \geq 0, \; a_{1,irreg(level)} \geq 0, \; a_{1,irreg(level)} + b_{1irreg(level)} < 1.$$

The properties of the LL model, when the disturbances follow GARCH(1,1) process, are presented in Pellegrini, Ruiz and Espasa (2007, 2008).

The last model is a simple AR(2)-SV model, which is used as a benchmark model. It has the following form:

$$\Delta y_t = \delta_0 + \delta_1 \Delta y_{t-1} + \delta_2 \Delta y_{t-2} + \varepsilon_t, \; \varepsilon_t \sim N\left(0, \sigma_t^2\right). \tag{10}$$

3. Forecasting Inflation

In this section, we examined whether the proposed models successfully predict the value of the inflation rate. We considered $T = 204$ monthly observations on the logarithm of CPI from January 1992 till December 2008. The data set employed in this study consists of logarithmic transformations of the original series CPI, computed as $y_t = 100\ln\left(CPI_t\right)$. All data has been seasonally adjusted, using the moving average method implemented in Eviews 6.

Before starting the analysis, we briefly describe the processes that influence inflation in Poland over the past twenty years. During the first years of 90's the Polish economy has been transformed to a market economy. Due to marketization and stabilization program there were deep economic and social changes including the elimination of the state control of prices and liberalizing trade, investment and capital flow (Fallenbuchl, 1994). In 1990-1992 the Polish economy was in an early stage of transition and inflationary processes visibly accelerated reaching 685.8 % in 1990. Therefore we begin our analysis from January 1992 to avoid the unusual effects of hyperinflation. In the years 1993-1997, monetary policy was focused on neutralizing the powerful inflationary forces discernible within the Polish economy and then to achieve a further reduction in inflation[1]. The next years (1998–2004) increased Poland's central bank independence and monetary policy was focused on maintaining price stability and preparation for integration with the EU. After 2004 the main goal of monetary policy was to achieve the Maastricht price stability criterion in the coming future. The data previously discussed are presented in Figure 1. The vertical line indicates the limit between sample and forecast-period.

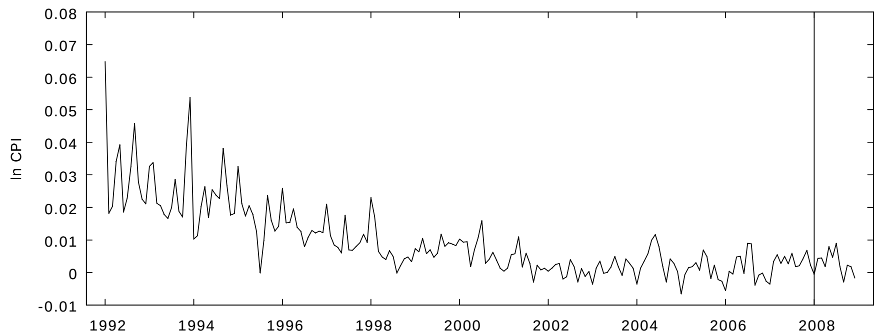

Figure 1. The values of logarithms of CPI (seasonally adjusted data). The vertical line indicates the limit between sample and forecast-period

Using Bayes' rule and Monte Carlo techniques, we calculated four competing Bayesian models – LL-SV, LL-GARCH(1,1), LL-GARCH(1,1)-Student and AR(2)-SV – based on dataset $y^{(t)}$ for $t = 1,...,192$. For each model and 12 months in 2008, as a result we obtained predictive distributions in the following form:

$$p\left(y_{192+h} \mid y^{(t)}, M_i\right), \text{ for } i = 1,...,4, \ h = 1,...,12, \qquad (11)$$

[1] NBP Annual Report, http://www.nbp.pl/

where LL-GARCH(1,1) – M_1, LL-GARCH(1,1)-Student – M_2, LL-SV – M_3 and finally AR(2)-SV – M_4.

Due to number of models and months, the entire procedure should be performed four times, giving a total of 48 predictive distributions.

Table 1. The quantiles of the predictive distribution

Date	Model			
	LL-GARCH(1,1)	LL-GARCH(1,1)-Student	LL-SV	AR(2)-SV
2008M01	0.3429	0.3535	0.3214	0.2825
[-0,0584]	(0.0367, 0.6564)	(0.1118, 0.6075)	(0.0894, 0.5398)	(0.0437, 0.5265)
2008M02	0.3429	0.3535	0.3214	0.4195
[0,4363]	(-0.0187, 0.7177)	(0.0791, 0.6499)	(0.0742, 0.5513)	(0.1303, 0.7037)
2008M03	0.3429	0.3535	0.3214	0.3364
[0,4509]	(-0.0662, 0.7587)	(0.0482, 0.6693)	(0.0520, 0.5732)	(0.0242, 0.6593)
2008M04	0.3429	0.3535	0.3214	0.3181
[0,1780]	(-0.0974, 0.8076)	(0.0160, 0.7123)	(0.0443, 0.5923)	(-0.0413, 0.6756)
2008M05	0.3429	0.3535	0.3214	0.3396
[0,8002]	(-0.1545, 0.8569)	(-0.0098, 0.7290)	(0.0329, 0.6017)	(-0.0584, 0.7361)
2008M06	0.3429	0.3535	0.3214	0.3399
[0,4702]	(-0.1728, 0.8888)	(-0.0348, 0.7577)	(0.0290, 0.6110)	(-0.0947, 0.7633)
2008M07	0.3429	0.3535	0.3214	0.3223
[0,8979]	(-0.2305, 0.9164)	(-0.0471, 0.7721)	(0.0128, 0.6237)	(-0.1432, 0.8031)
2008M08	0.3429	0.3535	0.3214	0.3221
[0,1819]	(-0.2603, 0.9458)	(-0.0671, 0.7965)	(-0.0159, 0.6441)	(-0.1724, 0.8319)
2008M09	0.3429	0.3535	0.3214	0.3245
[-0,2893]	(-0.2777, 0.9822)	(-0.0910, 0.8156)	(-0.0228, 0.6456)	(-0.1968, 0.8558)
2008M10	0.3429	0.3535	0.3214	0.3257
[0,2222]	(-0.3148, 1.0157)	(-0.1182, 0.8377)	(-0.0341, 0.6670)	(-0.2442, 0.8905)
2008M11	0.3429	0.3535	0.3214	0.3257
[0,1811]	(-0.3148, 1.0157)	(-0.1182, 0.8377)	(-0.0341, 0.6670)	(-0.2442, 0.8905)
2008M12	0.3429	0.3535	0.3214	0.3056
[-0,1699]	(-0.3944, 1.0672)	(-0.1724, 0.8619)	(-0.0571, 0.7093)	(-0.3194, 0.9236)

Note: All data are expressed as percentage.

Table 1 contains a selection of the corresponding out-of-sample forecasting results for the selected models with constant and time-varying mean. The data in brackets represent the seasonally adjusted consumer price index. For four models and for each month of 2008, the medians and the first and third quartiles (on the second line) of the predictive distributions are reported. A forecast will be said to be accurate if true value falls within the interquartile range. We can observe that during 2008 year the LL-GARCH(1,1) model produces the most accurate predictive distribution of CPI. In this case the true values fall within the interquartile range in ten cases of twelve. For the other models i.e. the LL-GARCH(1,1)-Student, LL-SV and AR(2)-SV, true values lie between the first and third quartiles only in eight cases of twelve. For the LL-GARCH(1,1) model, when inflation forecast is an accurate, the true values of CPI lie mostly between the median and third quartile. In other cases, predictive densities underestimate or overestimate the actual observation, both for the models with the constant and time-varying mean.

It is also worthwhile to check formally the accuracy of point forecasts of the mentioned above models. We measure forecast accuracy using two exact finite-sample test, namely: the sign test and Wilcoxon signed rank test (see Diebold and Mariano, 1995). These tests allow us to analyze of statistical differences between predictions generated by competing specifications when only a small number of forecasts are available. We test the null hypothesis of that the forecasting performance of the two different models is equally well (poor). The results are summarized in the Table 2, which includes p-values from the sign test (the first line) and Wilcoxon signed rank test (in the second line). In our case we use quadratic loss function. The observed loss differentials are free of serial correlation.

Table 2. Results from the sign test and Wilcoxon signed rank test

Model	Model			
	LL-GARCH(1,1)	LL-GARCH(1,1)-Student	LL-SV	AR(2)-SV
LL-GARCH(1,1)	-	0.7744 0.8501	1.0000 0.6772	1.0000 0.9697
LL-GARCH(1,1)-Student	-	-	0.7744 0.7910	1.0000 0.9697
LL-SV	-	-	-	0.7744 0.1514
AR(2)-SV	-	-	-	-

Note: The first line denotes p-values from the sign test. The second line includes p-values from the Wilcoxon signed rank test

According to the results given in the Table 2, we do not reject at conventional levels the hypothesis of equal expected quadratic loss. In other words there is no significant difference between the accuracy of point forecasts of the competing specifications. The standard AR(2)-SV model is not a significantly worse (better) predictor of the Polish CPI than the LL model.

It is well known, that Polish monetary authorities conduct the policy under inflation targeting regime, with the medium and long term target for CPI index fixed at 2.5% and with one percentage point of accepted deviation. Therefore it is interesting to consider what is the posterior probability of the hypothesis, that inflation will stay inside the targeting bound and how this posterior probability changes as the forecasts horizon grows. Bayesian methodology provides a direct way to predict different scenarios of inflation. The predictive distribution depicts the probability of various outcomes for CPI inflation in the future and allows to assess uncertainty of monetary policy. Unlike classical (sample-theory) approach we do not need carry out stochastic simulations since our approach follows from the basic rules of probability. Tables 3 and 4 present some characteristics of the probability distribution of the inflation path.

Table 3. Posterior probability that inflation will remain within the targeting regime

Date	Probability of inflation			
	LL-GARCH(1,1)	LL-GARCH(1,1)-Student	LL-SV	AR(2)-SV
	Within (1.5%; 3.5%)	within (1.5%; 3.5%)	within (1.5%; 3.5%)	Within (1.5%; 3.5%)
2008M01	0.0052	0.0052	0.0104	0.0052
2008M02	0.0833	0.0365	0.0417	0.0208
2008M03	0.2083	0.0833	0.0938	0.1198
2008M04	0.2656	0.1719	0.1771	0.2292
2008M05	0.3333	0.2240	0.2760	0.3177
2008M06	0.3385	0.2552	0.3021	0.3438
2008M07	0.4010	0.3333	0.4479	0.4063
2008M08	0.3854	0.2813	0.3438	0.3802
2008M09	0.3906	0.2865	0.3542	0.3750
2008M10	0.4167	0.3021	0.3958	0.3958
2008M11	0.4635	0.3385	0.4635	0.4323
2008M12	0.4688	0.3281	0.4635	0.4271

Table 4. Posterior probability that inflation will be above the upper bound of targeting regime

Date	Probability of inflation			
	LL-GARCH(1,1)	LL-GARCH(1,1)-Student	LL-SV	AR(2)-SV
	above 3.5%	above 3.5%	above 3.5%	above 3.5%
2008M01	0.9948	0.9948	0.9896	0.9948
2008M02	0.9167	0.9635	0.9583	0.9792
2008M03	0.7917	0.9167	0.9063	0.8802
2008M04	0.7344	0.8281	0.8229	0.7708
2008M05	0.6667	0.7760	0.7240	0.6823
2008M06	0.6615	0.7448	0.6979	0.6563
2008M07	0.5990	0.6667	0.5521	0.5938
2008M08	0.6146	0.7188	0.6563	0.6198
2008M09	0.6094	0.7135	0.6458	0.6250
2008M10	0.5833	0.6979	0.6042	0.6042
2008M11	0.5365	0.6615	0.5365	0.5677
2008M12	0.5313	0.6719	0.5365	0.5729

Tables 3 and 4 include assessment of the risks around central projections for prices of consumer goods and services. The predictive results indicate that inflation was more likely to be above target in 2008 than below target. The predictive probability for the hypothesis, that inflation will stay inside the targeting bound ranges from 0.0052 to 0.2656 in the period January–April, from 0.224 to 0.4479 in the period May–August and from 0.2865 to 0.4688 in the period September–December, whereas the predictive probability of the hypothesis that inflation will be above the upper limit of the tolerance band (3.5%), for all months and models, ranges from 0.5313 to 0.9948. These forecasts are consistent with true values of annual CPI because in 2008, according the GUS data[2], inflation rose above the upper limit of the tolerance band. During first eight months of 2008, CPI inflation showed a rising tendency – from 4.0% in January to 4.8% in August. In the period September – December we observed decline in the annual growth from 4.3% to 3.3%. According to reports published by NBP, the main factor conducing to higher level of inflation was the prices of energy commodities in the world market[3]. Thus, it seems that all models have ability to assess correctly the risk associated with CPI inflation.

4. Conclusions

In this paper the local level models are analyzed and compared from point of view of their ability to forecast monthly inflation in Poland. The data concern the consumer price index and they range from January 1992 till December 2008. For each model and for each month of 2008 we constructed the predictive distributions. According to the sign test and Wilcoxon signed rank test, there is no significant difference between the accuracy of point forecasts of the competing specifications. Also all models show correctly a rising tendency of annual CPI inflation. Analysis of forecast accuracy of competing specifications does not lead to decisive conclusion about superiority of any of the considered specifications. It seems that we can identify at least two reasons why unobserved component model could not satisfactorily predict the inflation.

Firstly, it is known that the first differences of local level model display the same correlation structure as the IMA(1,1) model – that is, one in which the first order autocorrelation is negative for the first difference of series and all other autocorrelations are zero (see West and Harrison, 1989). This is a very restrictive assumption which is in practice very difficult to obtain. From preliminary studies it was known that in our case not only first but also second order autocorrelation is negative and statistically significant, which may indicate a more complicated correlation structure of analyzed process. For this reason we consider a second order autoregressive process.

Secondly, the recent publication by Grassi and Proietti (2008) showed the strong evidence in favor of the local level model with heteroscedastic disturbances only in the core component of the U.S. inflation, whereas the transitory component was time invariant. The volatility of the disturbances driving only one i.e. core component may improve accuracy of forecasts.

[2] Central Statistical Office (GUS), http://www.stat.gov.pl/gus/index_ENG_HTML.htm

[3] Inflation Report, http://www.nbp.pl/

References

Bos, C. (2001), Time Varying Parameter Models for Inflation and Exchange Rates, WebDOC.

Diebold, F.X., Mariano, R.S. (1995), Comparing Predictive Accuracy, *Journal of Business and Economic Statistics*, 13, 253–63.

Durbin, J., Koopman, S.J. (2001), *Time Series Analysis by State Space Methods*, Oxford University Press, Oxford.

Fallenbuchl, Z.M. (1994), Foreign Trade in the Process of Transformation in Poland, *Atlantic Economic Journal*, 22, 2, 51–60.

Grassi, S., Proietti, T. (2008), Has the Volatility of U.S. Inflation Changed and How?, MPRA Paper 11453.

Harvey, A.C. (1989), *Forecasting, Structural Time Series Models and the Kalman Filter*, Cambridge University Press, Cambridge.

Koop, G. (2003), *Bayesian Econometrics*, John Wiley & Sons.

Koop, G., Potter, S. (2001), Are Apparent Findings of Nonlinearity Due to Structural Instability in Economic Time Series?*, The Econometrics Journal*, 4, 1, 37–55.

Muth, J.F. (1960), Optimal Properties of Exponentially Weighted Forecasts, *Journal of the American Statistical Association*, 55, 299–306.

Pellegrini, S., Ruiz, E., Espasa, A. (2007), The Relationship Between ARIMA-GARCH and Unobserved Component Models with GARCH Disturbances, Working Paper, ws072706.

Pellegrini, S., Ruiz, E., Espasa, A. (2008), ARIMA-GARCH and Unobserved Component Models with GARCH Disturbances: Are their Prediction Intervals Different? Job Market Paper.

Stock, J.H., Watson, M.W. (2007), Why has U.S. Inflation Become Harder to Forecast? *Journal of Money, Credit, and Banking*, 39, 3–33.

Stock, J.H., Watson, M.W. (2008), Phillips Curve Inflation Forecasts, Working Paper, 14322.

West, M., Harrison, J. (1989), *Bayesian Forecasting and Dynamic Models*, Springer.

Prognozowanie inflacji w Polsce przy użyciu modelu lokalnego poziomu

Z a r y s t r e ś c i. W artykule przeprowadzono badania dotyczące trafności prognoz otrzymanych za pomocą modelu lokalnego poziomu w wersji Stocka i Watsona (2008). Rozważono różne postacie tego modelu i zbadano, które z nich dają możliwość uzyskania najtrafniejszej prognozy. Badania empiryczne dotyczyły inflacji w Polsce w latach 1992-2008. Ostatni rok posłużył do oceny jakości prognoz. Badania przeprowadzono na podstawie wskaźnika cen konsumenta CPI. Uzyskane wyniki nie potwierdzają jednoznacznej przewagi modelu lokalnego poziomu, w prognozowaniu inflacji, nad standardowym modelem autoregresyjnym. Wszystkie modele uzyskały zadowalającą dokładność prognozy.

S ł o w a k l u c z o w e: model lokalnego poziomu, prognozowanie, inflacja.